The jamming/unjamming transitions

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One of the most important and elusive questions that lie in the interface of statistical physics and materials science is: how do systems transition **continuously** between fluid and solid states of matter. This phenomenon, coined as jamming or unjamming depending on whether the fluid $\rightarrow$ solid, or solid $\rightarrow$ fluid transition is considered, occurs both in equilibrium and driven systems, and is of immense technological importance, in addition to posing perplexing conceptual and methodological challenges to theoretical physicists and materials scientists.

In this mini-course, the problems of jamming and unjamming will be introduced by presenting some of the many instances of physical system in which they occur. Several recent theoretical approaches will be introduced and covered thoroughly.

The mini-course will be divided into three parts, as follows:

1. The unjamming transition in disordered solids. In this section we will study the nature of low-energy excitations in disordered solids. To this aim, both the continuum and the microscopic theories of linear elasticity will be covered. We will then look into the long-lasting problem of the Boson peak in glassy solids, by studying perhaps the simplest model of a glass, namely disordered networks of Hookean springs, and focusing on the elastic properties of this model. We will incrementally derive the density of vibrational modes in the framework of this simple model, and understand what are the key microscopic parameters and how these determine the frequency scales that characterize it. We will see how, at the unjamming point, the conditions of solidity are violated. Finally, we will study the singularities in elastic properties that accompany the unjamming transition, its associated diverging lengthscales, and the associated breakdown of continuum linear elasticity.

   Recommended literature:

   - *Dynamical heterogeneities in glasses, colloids, and granular media*, Editors: L. Berthier, G. Biroli, J.-P. Bouchaud, L. Cipeletti and W. van Saarloos Oxford University Press 2010.

   - *Geometric origin of excess low-frequency vibrational modes in weakly connected amorphous solids*, M. Wyart, S. R. Nagel, and T. A. Witten, Europhys. Lett. **72**, 486 (2005).

   - *Breakdown of continuum elasticity in amorphous solids*, E. Lerner, E. DeGiuli, G. Düring, and M. Wyart, Soft Matter **10**, 5085 (2014).

2. The role of marginal stability in the unjamming scenario. In this section we will start by considering the elasticity of packings of soft spheres close to the unjamming point. The principle of marginal stability will be introduced and applied to soft sphere packings, resulting in predictions for various scaling laws of their elastic properties. We will next consider the statistical mechanics of isostatic packings of hard spheres, and show how the principle of marginal stability can be applied to derive surprising relations between various structural properties. Finally, the scaling theory of entropic elasticity of colloidal glasses will be reviewed and related to the conventional elasticity of athermal soft sphere packings.

   Recommended literature:

   - *Effects of compression on the vibrational modes of marginally jammed solids*, M. Wyart, L. Silbert, S. R. Nagel, and T. Witten, Phys. Rev. E, **72**, 051306 (2005).

   - *Low-energy non-linear excitations in sphere packings*, E. Lerner, G. Düring, and M. Wyart, Soft Matter **9**, 8252 (2013).

   - *The distribution of forces affects vibrational properties in hard sphere glasses*, E. DeGiuli, E. Lerner, C. Brito, and M. Wyart, PNAS **111**, 17054 (2014).

3. The jamming transition of overdamped floppy networks and granular suspensions. In this section we will review the theory of the mechanics of overdamped floppy networks of rigid rods. We will show that a framework can be set up for these floppy networks, that is dual to the framework of the elasticity of spring networks. In this dual framework, objects analogous to low-energy elastic excitations can be defined and their statistics can be studied. Using this framework, we will study the mechanics of floppy networks of rigid bars under external deformations, and make a connection with the mechanics of biological networks. We will then relate the mechanics of floppy networks to the rheology of non-Brownian granular suspensions. The properties of isostatic packings of hard spheres will be shown to be related to rheological observables of non-Brownian granular suspensions away from the jamming critical point.

Recommended literature:

- *Phonon gap and localization lengths in floppy materials*, G. Düring, E. Lerner, and M. Wyart, Soft Matter **9**, 146 (2013).

- *Length scales and self-organization in dense suspension flows*, G. Düring, E. Lerner, and M. Wyart, Phys. Rev. E **89**, 022305 (2014).

- *A unified framework for non-Brownian suspension flows and soft amorphous solids*, E. Lerner, G. Düring, and M. Wyart, PNAS **109**, 4798 (2012).