

# Quasi normal modes

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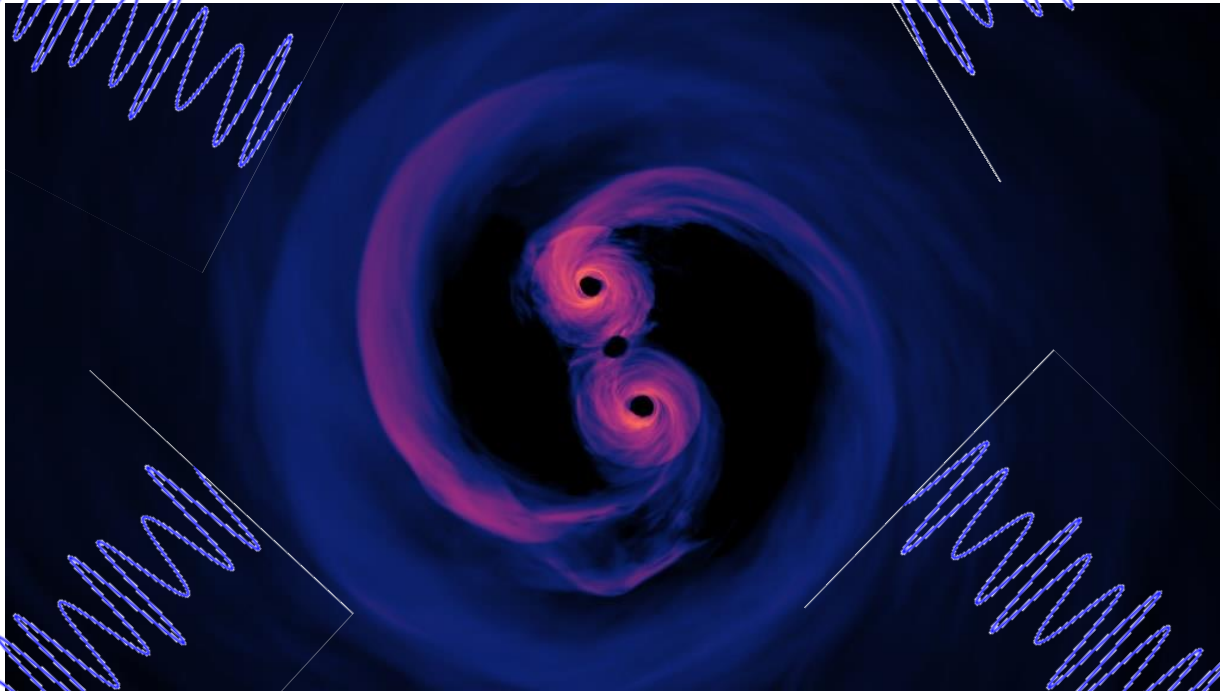
30 May 2023

Radboud University

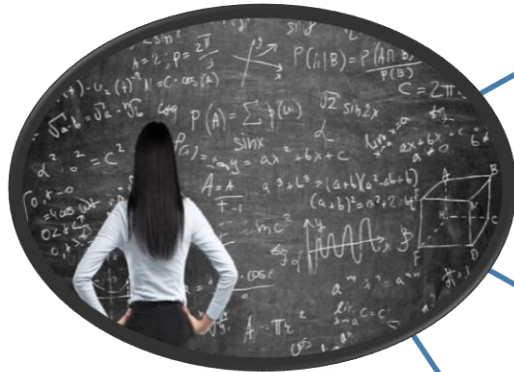


# Two of my favorite topics

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# Various expansion parameters



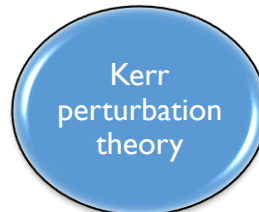
- Background: Minkowski
- Parameter:  $G$
- Application: Scattering amplitudes, EFT, often combined with post-Newtonian expansions



- Background: Minkowski
- Parameter:  $1/c^2$
- Application: GWs during inspiral



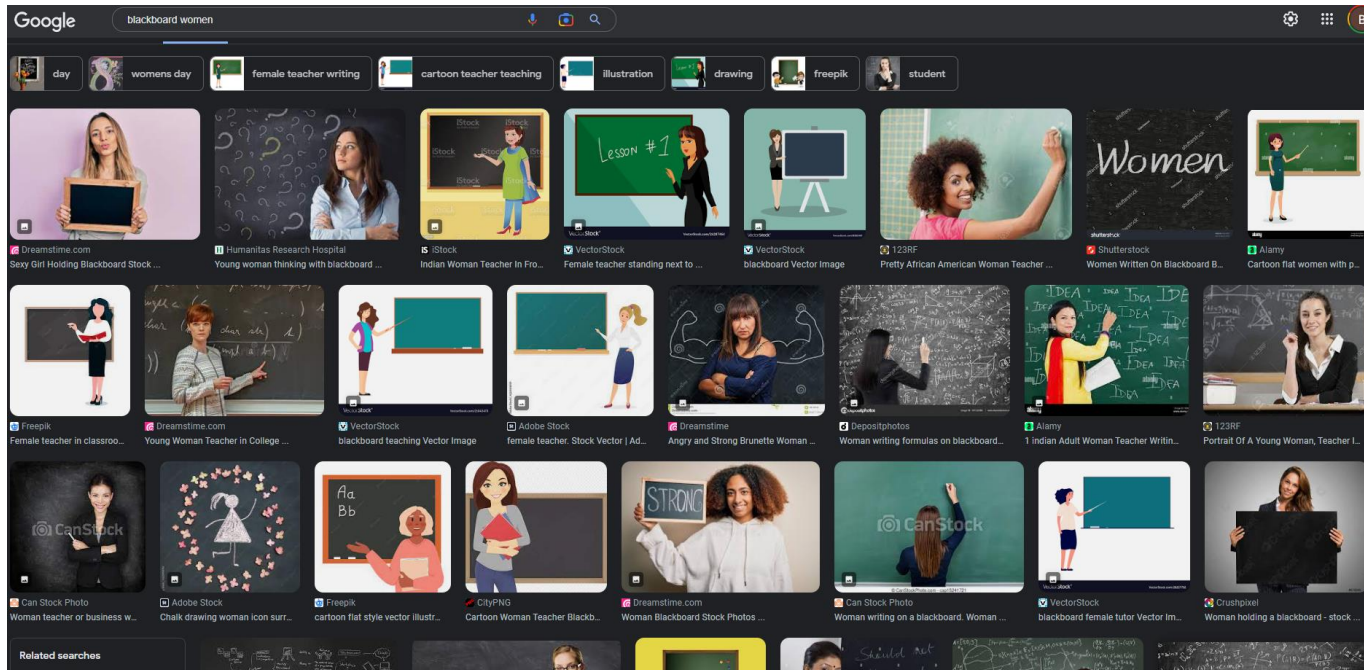
- Background: Schwarzschild
- Parameter: often perturbing mass  $m$
- Application: EMRIs, QNMs



- Background: Kerr
- Parameter: often perturbing mass  $m$
- Application: EMRIs, QNMs



# Random lesson: don't Google blackboard+women





# Blackboard+men

Google blackboard men

GoodFon Wallpaper man, science, blackboard ...

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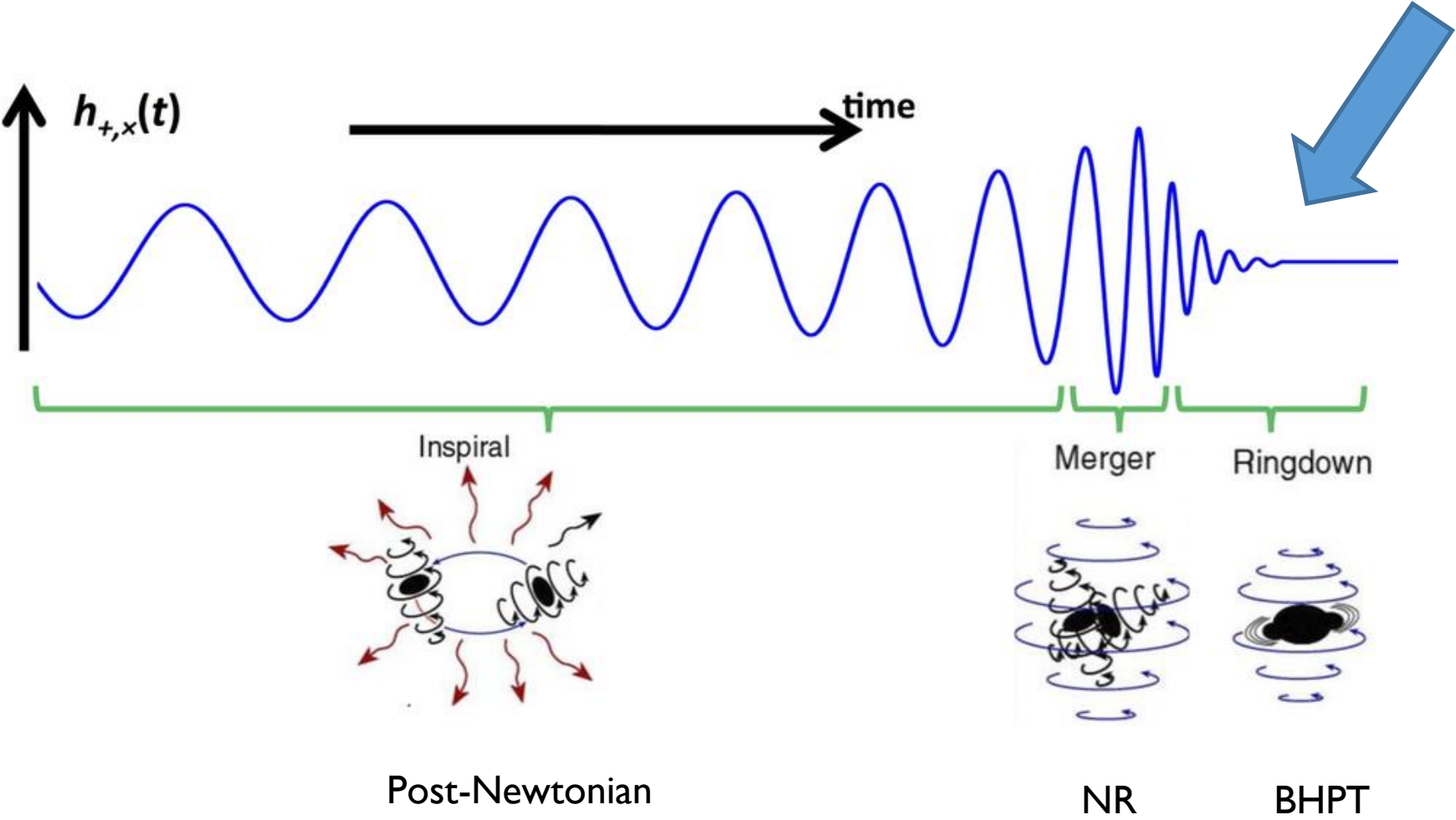
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science wallpaper

# Gravitational waves from black hole mergers



# Quasi-normal modes

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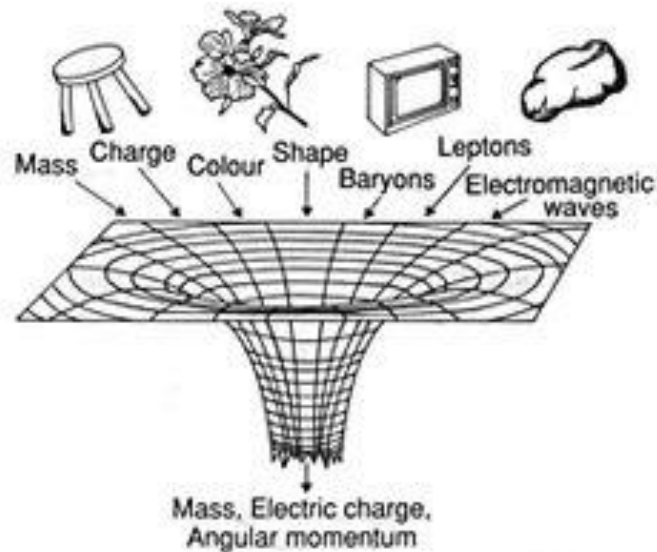
# Black holes are the simplest bells

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QNMs depend only on the Mass and Spin.

## No hair theorem

“No matter how black hole is formed or what you are throwing in, the resulting spacetime only depends on two parameters!”





# How to compute these frequencies?

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1. Split your metric  $g_{\mu\nu} = g_{\mu\nu}^{Kerr} + \epsilon h_{\mu\nu}$
2. Substitute in Einstein's equations and only keep terms linear in  $\epsilon$
3. Use clever variables and gauge choices to write the equation as a ***time-independent Schrodinger equation***
4. Solve for the frequency

# Scalar field on Schwarzschild

$$g_{\mu\nu}dx^\mu dx^\nu = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Start with the wave equation

$$\begin{aligned}\nabla_\mu \nabla^\mu \varphi &= \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi) = 0 \\ \frac{1}{r^2 \sin \theta} \partial_t \left( -\frac{r^2 \sin \theta}{f} \partial_t \varphi \right) &+ \frac{1}{r^2 \sin \theta} \partial_r (r^2 \sin \theta f \partial_r \varphi) \\ &+ \frac{1}{r^2} D^2 \varphi = 0\end{aligned}$$

Decompose into spherical harmonics

$$\varphi(t, r, \theta, \phi) = \frac{1}{r} \sum_{l,m} \varphi_{lm}(t, r) Y_{lm}(\theta, \phi)$$

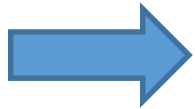
Substitute and multiply by  $r f(r)$

$$-\partial_t^2 \varphi_{lm} + f \partial_r (f \partial_r \varphi_{lm}) - \underbrace{f(r) \left( \frac{l(l+1)}{r^2} + \frac{\partial_r f}{r} \right)}_{= V_l^{\text{scalar}}} \varphi_{lm} = 0$$

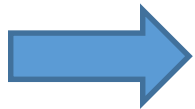
## $r$ versus $r_*$

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$$\begin{aligned}g_{\mu\nu}dx^\mu dx^\nu &= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \\ &= f(-dt^2 + dr_*^2) + r^2(d\theta^2 + \sin^2\theta d\phi^2)\end{aligned}$$



$$dr_* = \frac{dr}{1 - \frac{2M}{r}}$$



$$r_* = r + 2M \ln\left(\frac{r}{2M} - 1\right)$$



# Simplicity in terms of tortoise coordinate

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Note that  $f \partial_r (f \partial_r \varphi_{lm}) = \partial_{r_\star}^2 \varphi_{lm}$

so that we obtain

$$-\partial_t^2 \varphi_{lm} + \partial_{r_\star}^2 \varphi_{lm} - V_l(r) \varphi_{lm} = 0$$

Cost: two different radii



# Frequency space

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$$-\partial_t^2 \varphi_{lm} + \partial_{r_*}^2 \varphi_{lm} - V_l(r) \varphi_{lm} = 0$$

Go to Fourier space

$$\varphi_{lm}(t, r) = \int \frac{d\omega}{\sqrt{2\pi}} \tilde{\varphi}_{lm}(\omega, r) e^{i\omega t}$$

to find a time-independent Schrödinger equation

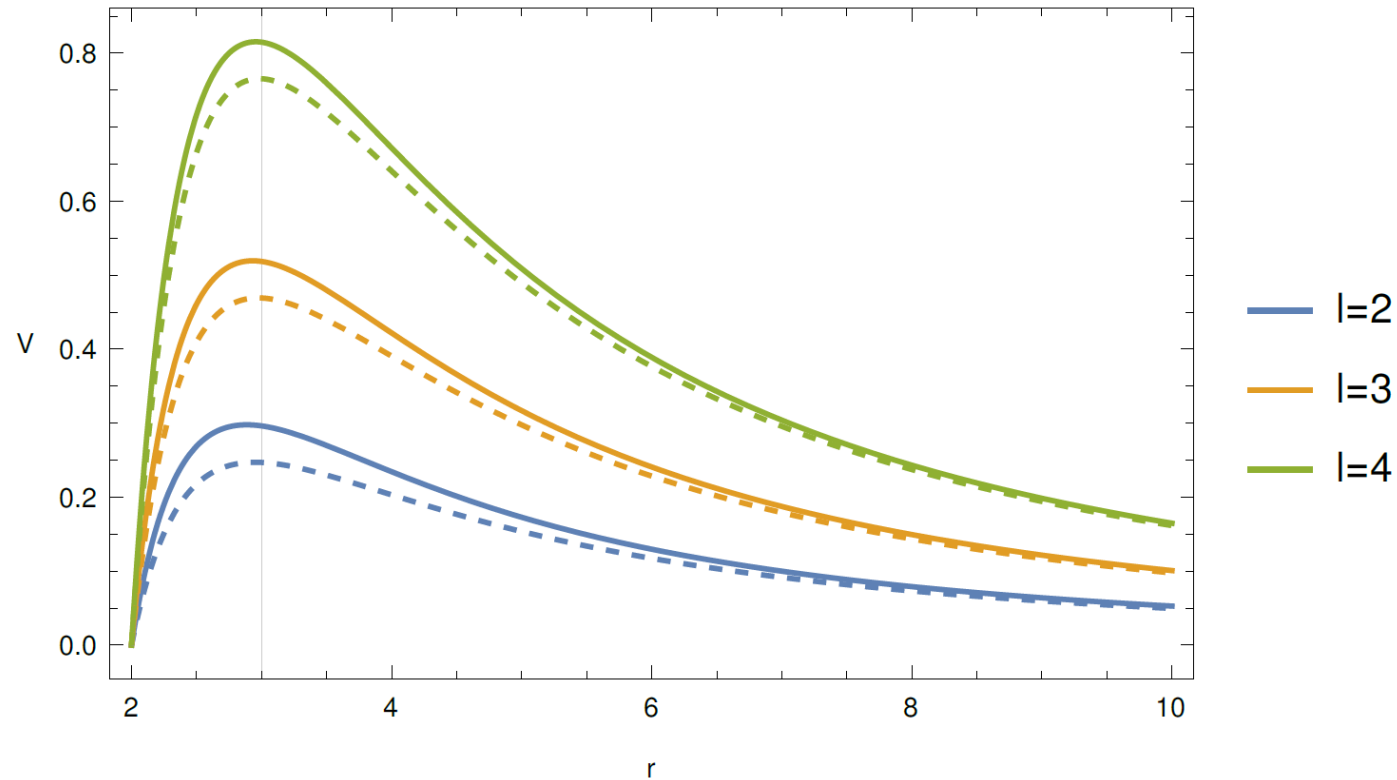
$$\partial_{r_*}^2 \tilde{\varphi}_{lm} + (\omega_{lm}^2 - V_l(r)) \tilde{\varphi}_{lm} = 0$$





# Potentials for scalar and gravitational case

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$$V_l^{scalar}(r) = f(r) \left( \frac{l(l+1)}{r^2} + \frac{2M}{r} \right)$$

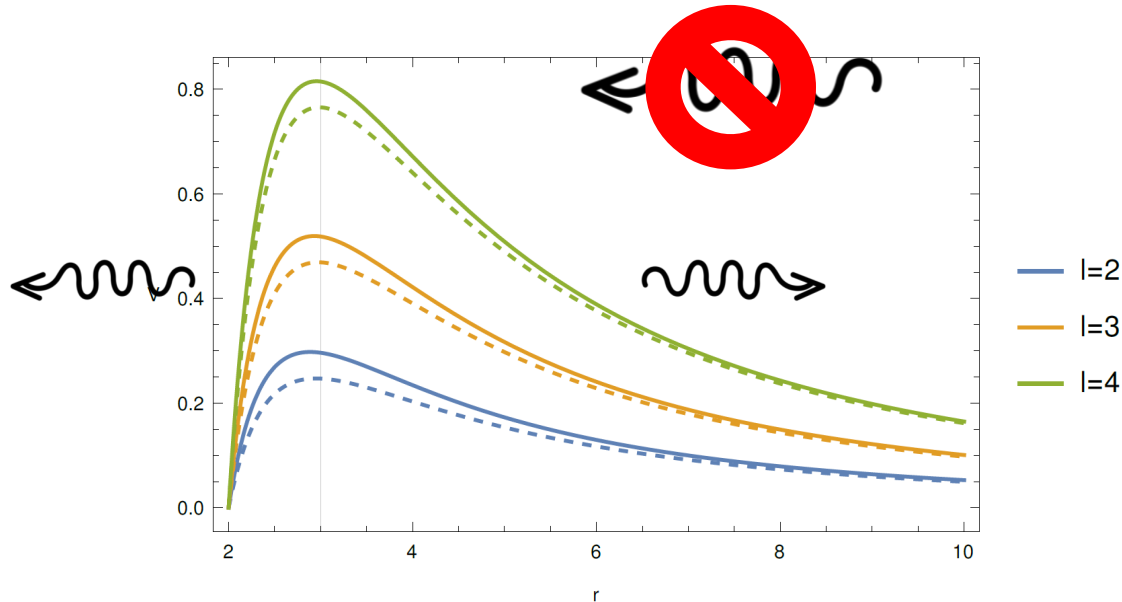
$$V_l^{odd}(r) = f(r) \left( \frac{l(l+1)}{r^2} + \frac{6M}{r} \right)$$

# Boundary conditions: “free” oscillation

$$\partial_{r_\star}^2 \tilde{\varphi}_{lm} + (\omega_{lm}^2 - V_l(r)) \tilde{\varphi}_{lm} = 0$$

QNMs are solutions with

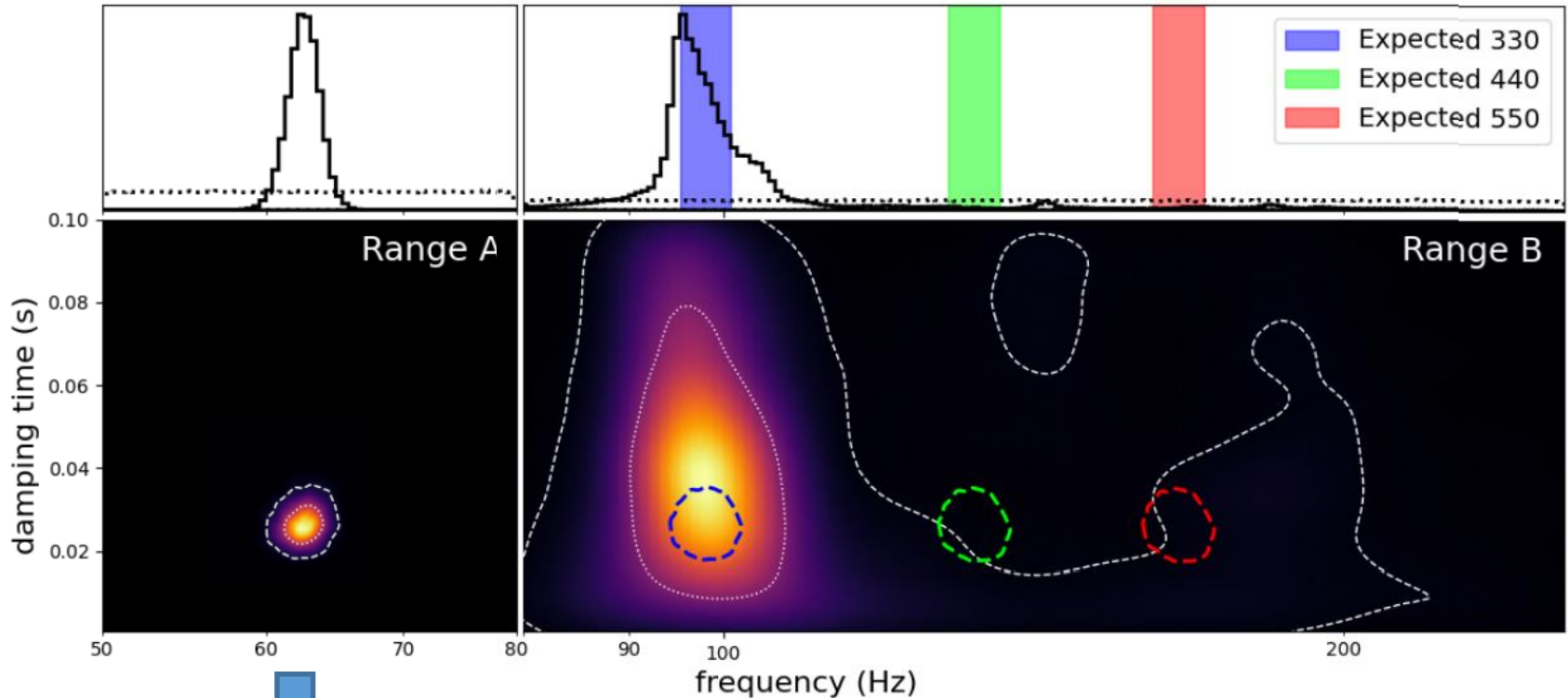
$$\tilde{\varphi}_{lm} = \begin{cases} A_t(\omega) e^{i\omega r_\star} + A_r(\omega) e^{-i\omega r_\star} & r_\star \rightarrow \infty \\ A_t(\omega) e^{i\omega r_\star} & r_\star \rightarrow -\infty \end{cases}$$



Can we observe QNMs?

# Black hole spectroscopy

GW190521: a massive asymmetric merger



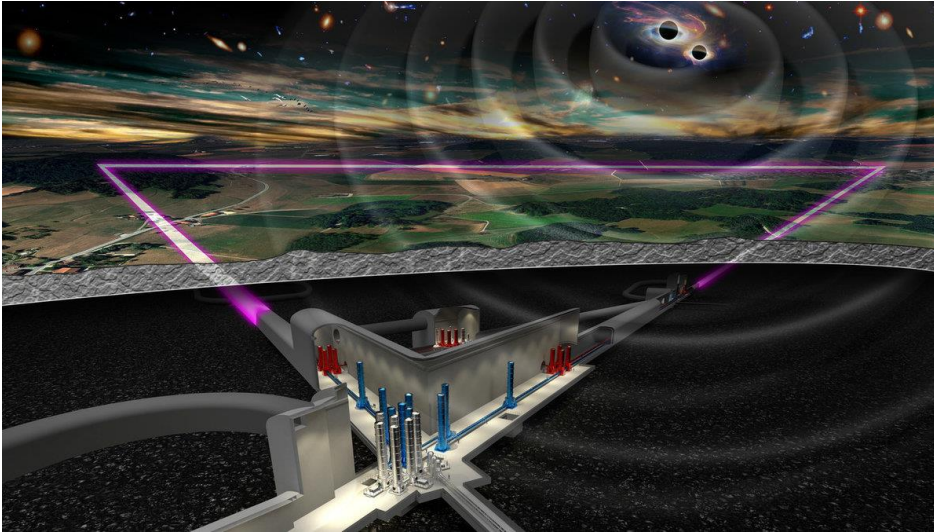
Assume fundamental mode:

Mass and spin BH

# The future

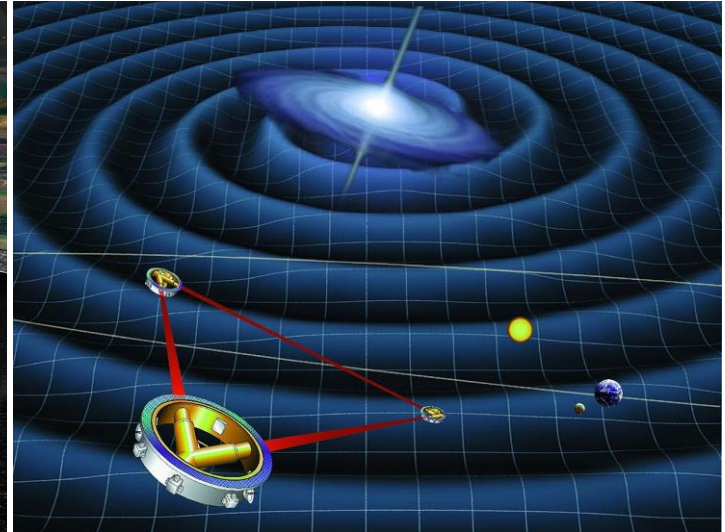
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$1 - 10^4$  Hz



Einstein Telescope  
(or Cosmic Explorer)

$10^{-4} - 1$  Hz

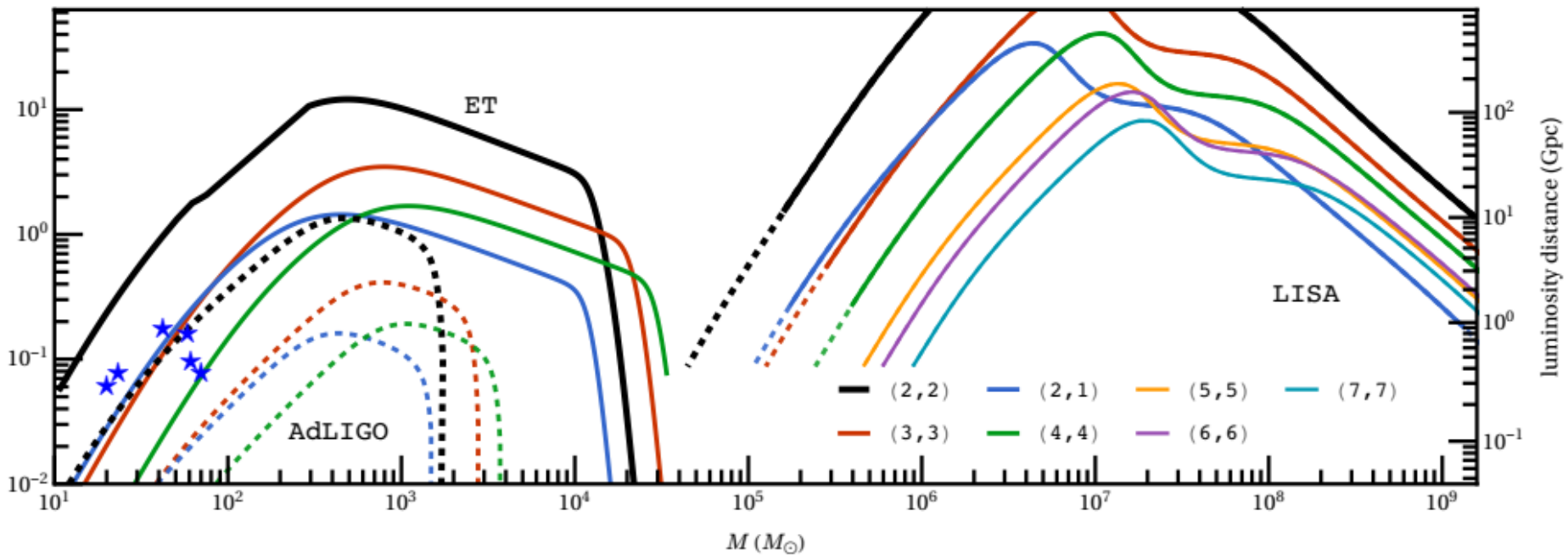


LISA  
(or TianQin)



# The future

Forecast for non-spinning binary with  $m_1 = 2m_2$



# Quasi normal modes are...

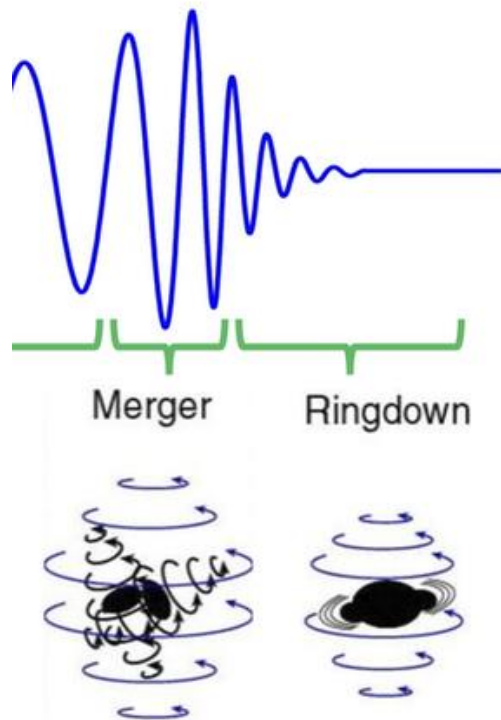
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1. Awesome
2. “Free” oscillations of a black hole
3. Completely determined by the mass and spin of the final black hole
4. Observable
5. New ways to test the nature of black holes and General Relativity

# Non-linearities

# When is QNM description valid?

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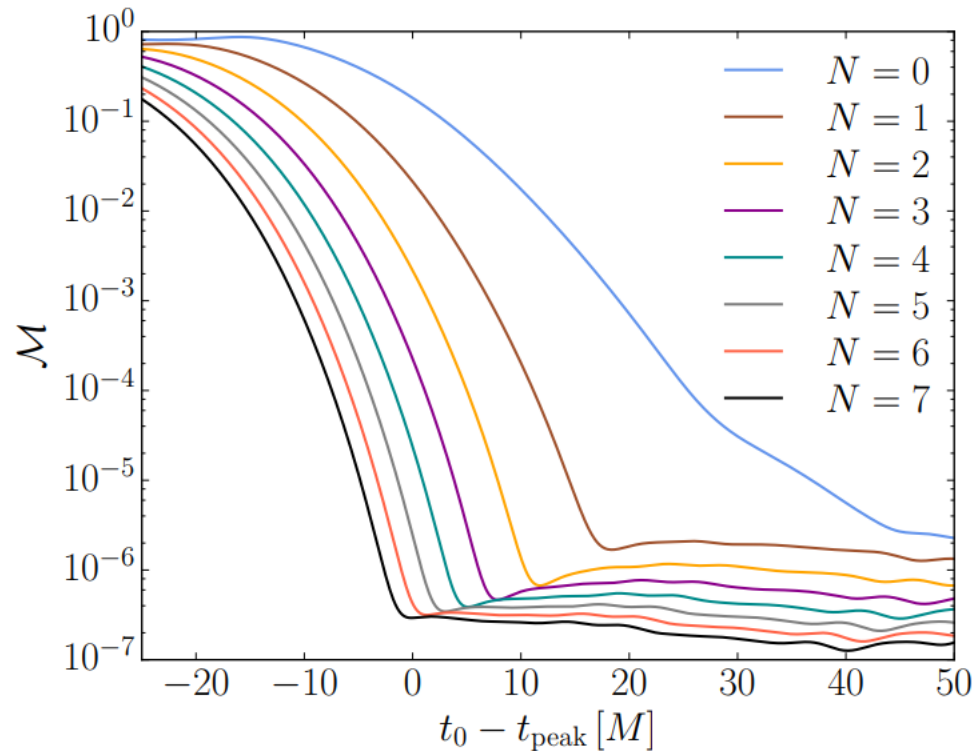
Ringdown starts when...

- $h_+/h_\times$  has a peak?
- $\Psi_4$  has a peak?
- there is a common (apparent) horizon?
- $10M$  after any of these?
- or  $20M$ ?
- ....?

# At the peak already?

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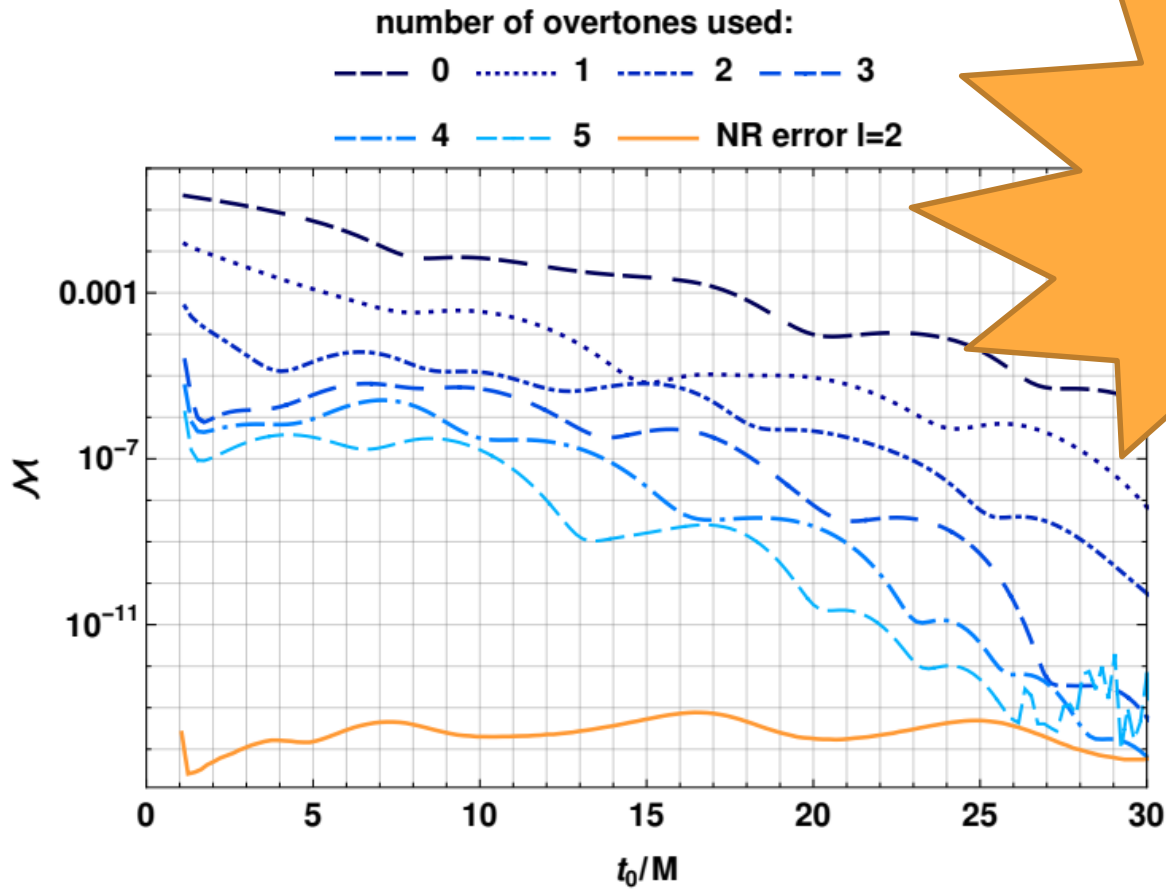
Observation: many overtones  $\rightarrow$  lower mismatch



[Giesler et al, 2019]



# Similar results at the horizon



Horizon is in  
strong field  
regime!

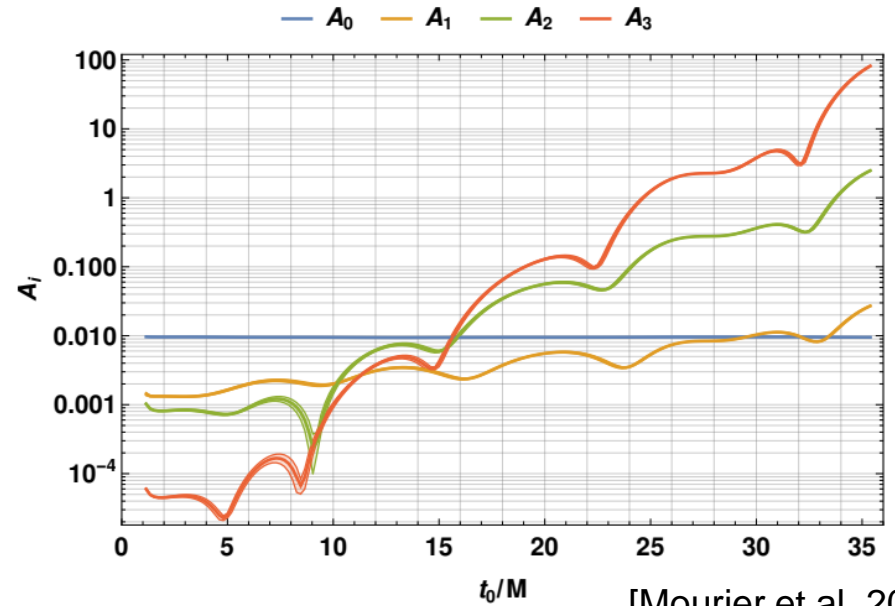
[Mourier et al, 2020]

# Conclusion: matter of overfitting

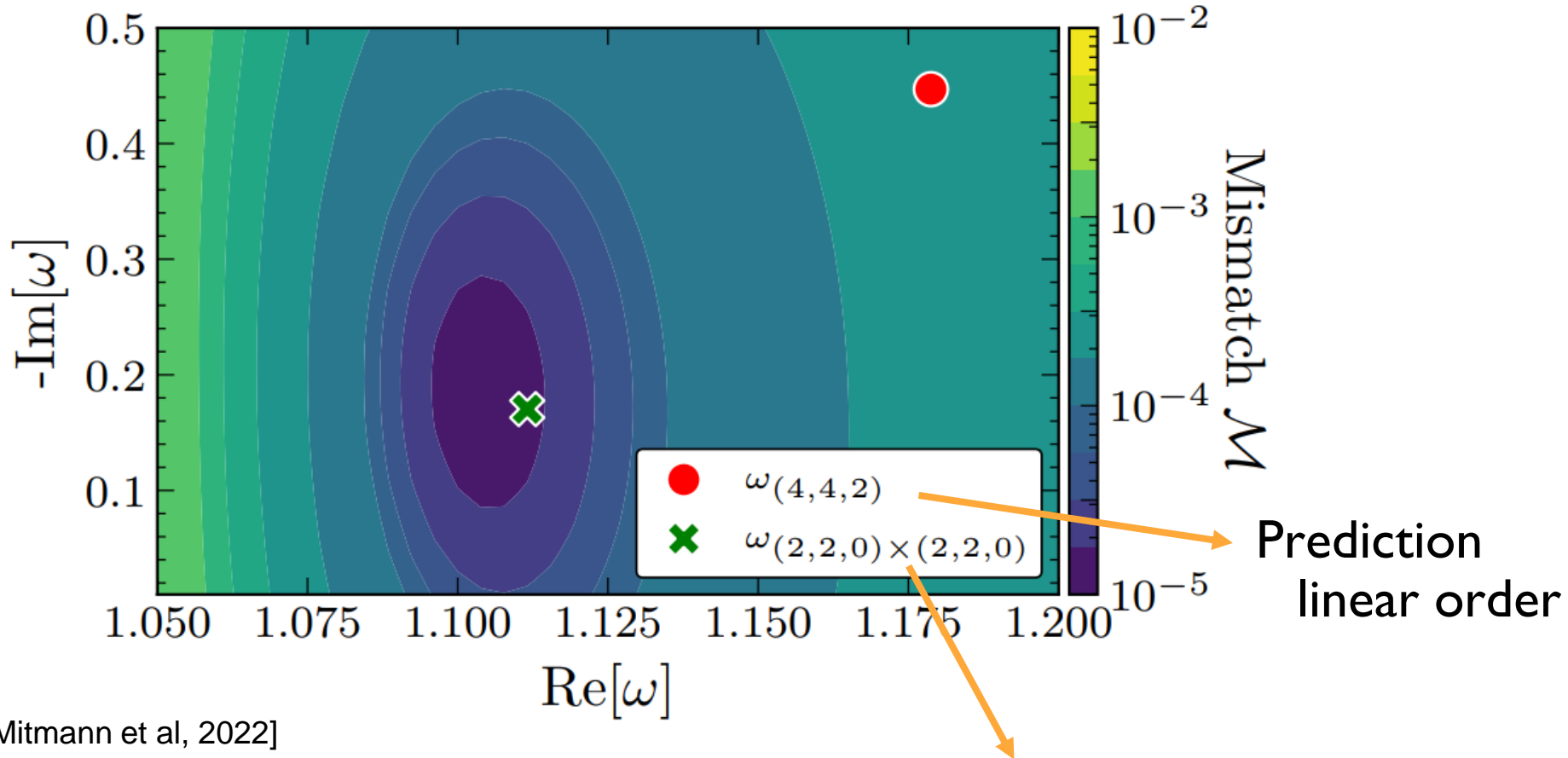
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$$h_{lm}(t, r) = \frac{1}{r} \sum_{n=0}^N A_{lmn} e^{-i\omega_{lmn}(t-t_0) + \Phi_{lmn}}$$

By construction = constants,  
but fits show..



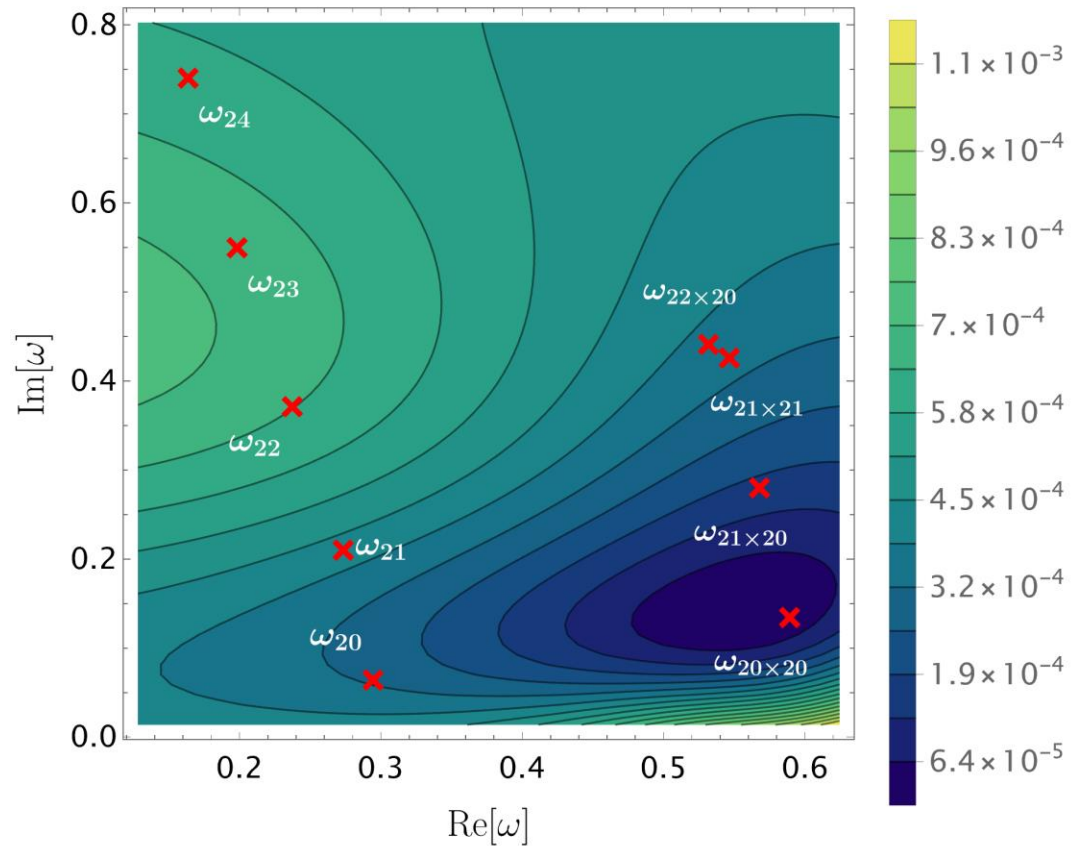
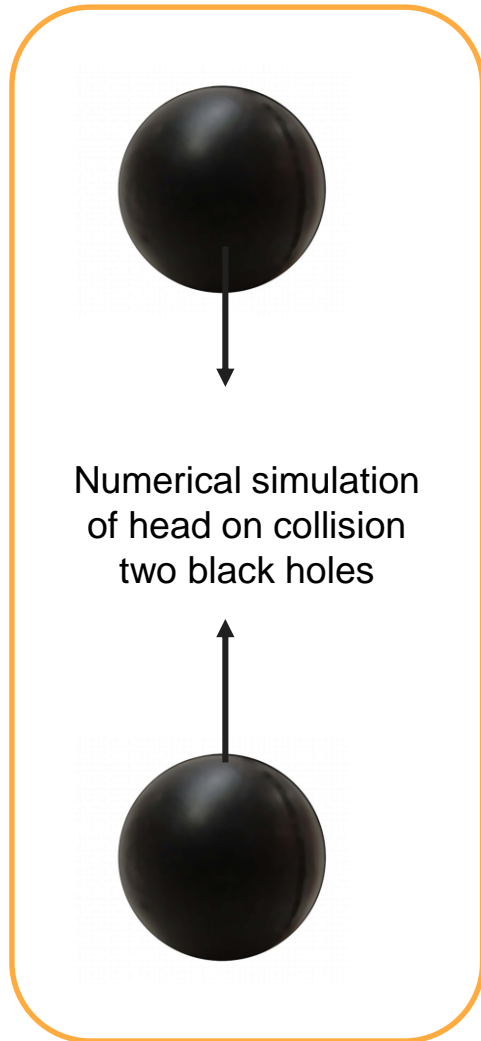
# Non-linear model preferred @ infinity



[Mitmann et al, 2022]

Prediction  
quadratic order  
in perturbation  
theory

# Also true @ black hole horizon



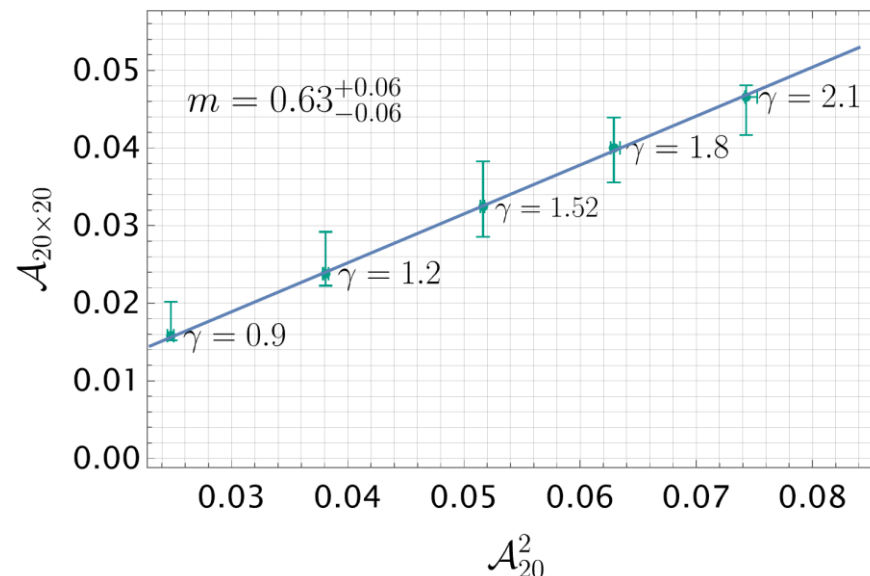
# But surprisingly difficult to find!

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$$A_{l_1 m_1 \times l_2 m_2} = \alpha A_{l_1 m_1} A_{l_2 m_2}$$

Two strategies

1. Use boosted black holes to enhance the linear signal
2. Use very accurate numerical simulations





# Take home message

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Enjoy the rest of this school!

...and remember quasi-normal modes are awesome (and if you want to know more: ask Ariadna)



Back up slides

# Mathematical description

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$$h_+ + i h_\times = \sum_{l=2}^{\infty} \sum_{m=-l}^l h_{lm}(t, r) {}_2Y_{lm}(\theta, \phi)$$

spin-weighted  
spherical harmonic

$$h_{lm}(t, r) = \frac{1}{r} \sum_{n=0}^N A_{lmn} e^{-i\omega_{lmn}(t-t_0) + \Phi_{lmn}}$$

Depend on the details  
of the “hammer”

# Frequencies and damping times

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$$\omega_{lmn} = \omega_{lmn}^R + i \omega_{lmn}^I = 2\pi f_{lmn} + \frac{i}{\tau_{lmn}}$$



Depends on three integers:

$$l = 2, 3, \dots$$
$$-l < m < l$$
$$n = 0, 1, 2, \dots$$

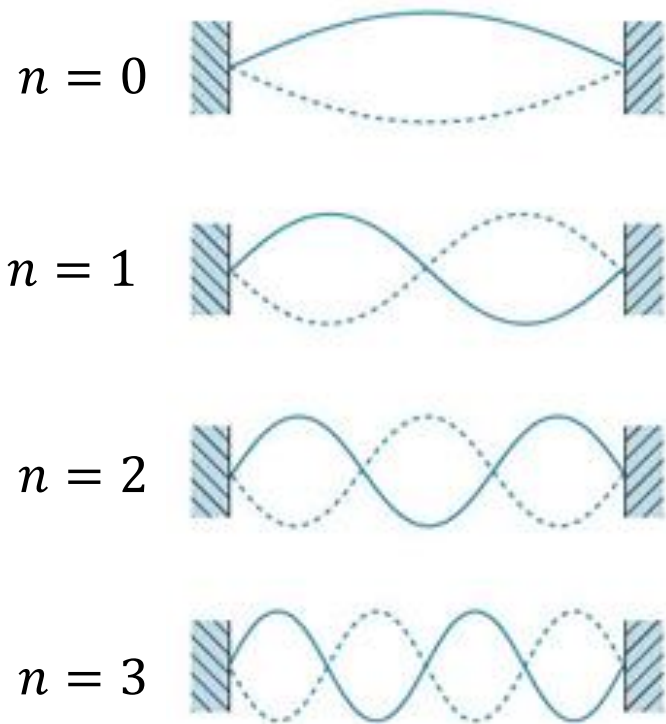


Damping time

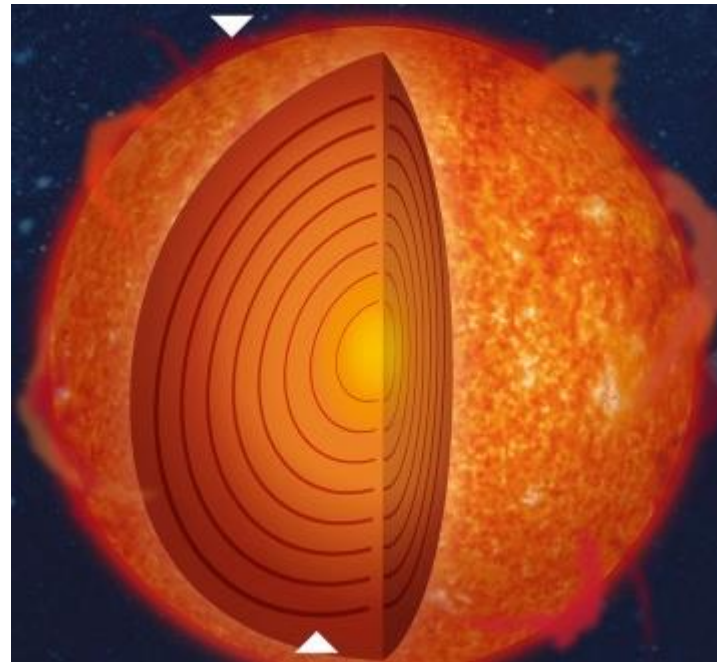
# Normal modes: no dissipation

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String



Stars



*asteroseismology*