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Certifying spectral gaps of quantum many-body systems

Abstract: In this presentation, I will explore the challenges and methodologies associated with proving the presence of spectral gaps in quantum many-body systems, with a focus on the 1D AKLT model. Spectral gaps are central in our understanding of physical properties, such as ground state structure and correlation laws. In addition, spectral gaps are of direct importance in quantum computing, as they indicate how efficient an adiabatic quantum computation can be with provable guarantees of success.

We introduce two innovative constructions based on semidefinite programming (SDP) aimed at local, frustration-free, and translation-invariant Hamiltonians. The first employs positive operator decompositions to obtain translation invariant operators, enhancing the accuracy of the gap's lower bound with increased operator support size. The second utilizes the moment matrix method for a tighter lower bound through sum-of-squares decompositions involving non-commuting variables represented as Pauli operators.

Furthermore, I will discuss the role of SU(2) symmetries in simplifying the problem via Schur-Weyl duality, leading to significant computational efficiency improvements. This approach not only advances our understanding of quantum systems but also serves as a precursor for practical applications, such as the adiabatic preparation of tensor network states, by providing a reliable method to certify spectral gaps.